CMU 15-671 Models of Software Systems Fall 1995

Sets, Relations, Functions; Proof Techniques

Garlan & Wing Homework 3 Due: 13 September 1995

Problem 1.
Chapter 5 of SEM: 5.1, 5.2, 5.4, 5.6, 5.9, 5.13

Problem 2.
Chapter 6 of SEM: 6.1, 6.2, 6.3, 6.11

Problem 3.
Simplify this expression using the One-Point Rule.

$$\exists\, names : ID \rightarrowtail PERSON \bullet \exists\, enemies : \mathbb{P}\, ID \bullet enemies \subseteq dom\ names \wedge names = \varnothing \wedge enemies = \varnothing$$

Problem 4.
[GS, Chapter 4, Problem 4.10]
Prove Modus ponens, $p \wedge (p \Rightarrow q) \Rightarrow q$, using equational reasoning. Hint: You should also use one of the proof techniques discussed in Handout 3.

Problem 5.
[GS, Chapter 4, Problem 4.13]
Let $x \downarrow y$ be the minimum of integers x and y, defined by $x \downarrow y = (\textbf{if } x \leq y \textbf{ then } x \textbf{ else } y)$. Prove that $\downarrow$ is symmetric, i.e., $b \downarrow c = c \downarrow b$. Use an equational reasoning style of proof. How many cases do you have to consider? You may use the necessary rules of integer arithmetic, for example, that $b \leq c \equiv b = c \vee b < c$ and that $b < c \equiv c > b$.