Outline

- FOIL
- Induction as inverted deduction

Emergency C-Section Study

- Probability of emergency c-section is 0.12 over entire population of 10,000 patients
- Each patient described by 215 attributes
- Cost-benefit analysis suggests if probability of emergency c-sec > 0.4, then elective c-section improves the expected outcome.

Typical learned rule:

```
If   No previous vaginal deliveries, and
     Abnormal 2nd Trimester Ultrasound, and
     Malpresentation at admission
Then Probability of Emergency C-Section is 0.6

 Training set: 26/41 = .634
 Test set: 12/20 = .600
```

A Learned Text Extraction Rule

[Freitag, Mitchell]

A rule for location

```
If a word is parsed with a left J-link;
   and it is J-linked to ``in'';
   and it is not capitalized;
   and ``in'' has a left MV-link
Then it is the main location word.
```

A sentence it matches

A talk will follow in Hammerschlag Hall $\boxed{1112}$.

Parser output

```
 +---------------------Xp------------------------+
 +-Wd---+                   +--------Js------+  |
 | +-Ds-+--Ss--+--I--+-MVp--+    +--G---+--G-+  |
 | |    |      |     |      |    |      |    |  |
// A talk.n will.v follow.v in Hschlg Hall 1112 .
```

FOIL($Target\_predicate, Predicates, Examples$)

- $Pos \leftarrow$ positive $Examples$
- $Neg \leftarrow$ negative $Examples$
- while Pos, do

  Learn a NewRule

  - $NewRule \leftarrow$ most general rule possible
  - $NewRuleNeg \leftarrow Neg$
  - while $NewRuleNeg$, do

    Add a new literal to specialize NewRule

    1. $Candidate\_literals \leftarrow$ generate candidates
    2. $Best\_literal \leftarrow$
       $\text{argmax}_{L \in Candidate\_literals} Foil\_Gain(L, NewRule)$
    3. add $Best\_literal$ to $NewRule$ preconditions
    4. $NewRuleNeg \leftarrow$ subset of $NewRuleNeg$
       that satisfies $NewRule$ preconditions
  - $Learned\_rules \leftarrow Learned\_rules + NewRule$
  - $Pos \leftarrow Pos - \{$members of Pos covered by $NewRule\}$
- Return $Learned\_rules$

Specializing Rules in FOIL

Learning rule: $P(x_1, x_2, \ldots, x_k) \leftarrow L_1 \ldots L_n$
Candidate specializations add new literal of form:

- $Q(v_1, \ldots, v_r)$, where at least one of the v_i in the created literal must already exist as a variable in the rule.
- $Equal(x_j, x_k)$, where x_j and x_k are variables already present in the rule
- The negation of either of the above forms of literals

Information Gain in FOIL

$$Foil\_Gain(L,R) \equiv t\left(\log_2 \frac{p_1}{p_1+n_1} - \log_2 \frac{p_0}{p_0+n_0}\right)$$

Where

- L is the candidate literal to add to rule R
- $p_0 =$ number of positive bindings of R
- $n_0 =$ number of negative bindings of R
- $p_1 =$ number of positive bindings of $R+L$
- $n_1 =$ number of negative bindings of $R+L$
- t is the number of positive bindings of R also covered by $R+L$

Note

- $-\log_2 \frac{p_0}{p_0+n_0}$ is optimal number of bits to indicate the class of a positive binding covered by R

Induction as Inverted Deduction

Induction is guessing h such that

$$(\forall x_i)\ B \wedge h \wedge x_i \vdash f(x_i)$$

where

- x_i is ith training instance
- $f(x_i)$ is the target function value for x_i
- B is other background knowledge

Induction as Inverted Deduction

"pairs of people, $\langle u, v \rangle$ such that child of u is v,"

$$\begin{aligned} f(x_i) : & \quad Child(Bob, Sharon) \\ x_i : & \quad Male(Bob), Female(Sharon), Father(Sharon, Bob) \\ B : & \quad Parent(u, v) \leftarrow Father(u, v) \end{aligned}$$

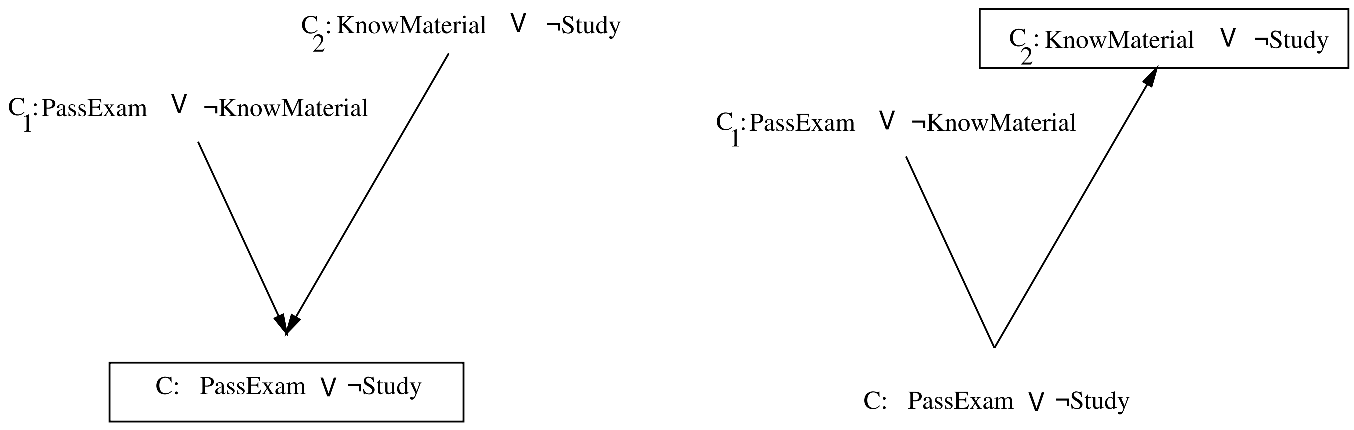
What satisfies $(\forall x_i)\ B \wedge h \wedge x_i \vdash f(x_i)$?

$$\begin{aligned} h_1 : & \ Child(u, v) \leftarrow Father(u, v) \\ h_2 : & \ Child(u, v) \leftarrow Parent(u, v) \end{aligned}$$

Deduction: Resolution Rule

$$\frac{\begin{array}{rcl} P & \vee & L \\ \neg L & \vee & R \end{array}}{\begin{array}{rcl} P & \vee & R \end{array}}$$

Inverting Resolution



Inverting Resolution

1. Given initial clauses C_1 and C_2, find a literal L from clause C_1 such that $\neg L$ occurs in clause C_2
2. Form the resolvent C by including all literals from C_1 and C_2, except for L and $\neg L$. More precisely, the set of literals occurring in the conclusion C is

   $$C = (C_1 - \{L\}) \cup (C_2 - \{\neg L\})$$

   where $\cup$ denotes set union, and "$-$" denotes set difference.

First order resolution

First order resolution:

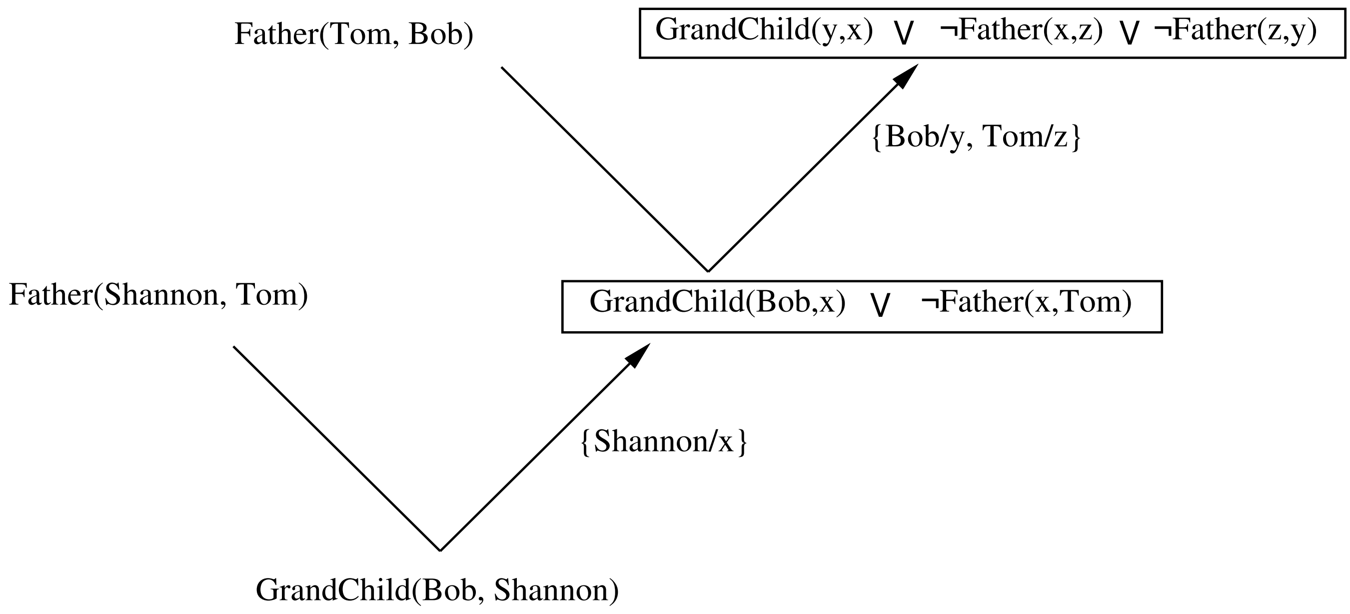
1. Find a literal L_1 from clause C_1, literal L_2 from clause C_2, and substitution θ such that $L_1\theta = \neg L_2\theta$
2. Form the resolvent C by including all literals from $C_1\theta$ and $C_2\theta$, except for $L_1\theta$ and $\neg L_2\theta$. More precisely, the set of literals occurring in the conclusion C is

$$C = (C_1 - \{L_1\})\theta \cup (C_2 - \{L_2\})\theta \tag{1}$$

Inverting First order resolution

$$C_2 = (C - (C_1 - \{L_1\})\theta_1)\theta_2^{-1} \cup \{\neg L_1 \theta_1 \theta_2^{-1}\}$$

Cigol


Father(Tom, Bob)
GrandChild(y,x) ∨ ¬Father(x,z) ∨ ¬Father(z,y)
{Bob/y, Tom/z}
Father(Shannon, Tom)
GrandChild(Bob,x) ∨ ¬Father(x,Tom)
{Shannon/x}
GrandChild(Bob, Shannon)