# Tethers for Space Propulsion Without Reaction Mass

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## Abstract

The limitations of reaction-mass space propulsion make it worthwhile to consider means of space propulsion which do not require reaction mass. Tethers, long cables attached to satelites in orbit, can be used in two ways for propulsion. They can move around orbital momentum from one body to another, recovering orbital energy which would otherwise be wasted. Conducting tethers with electrical power sources can raise their orbits by interacting with planetary magnetic fields. The dynamics of tethers can be approximated by the mass dipole, which has a stable equilibrium in the orientation necessary for the two means of propulsion.

*Chapter 1*

# Rationale for Non-Reaction-Mass Propulsion

Current space propulsion uses chemical rockets[1], which are extremely expensive and are reaching the limits of their development. Thus, there is a great deal of interest in developing new propulsion systems. A variety of advanced rockets have been proposed, several of which accelerate their reaction mass with electric or magnetic fields: Ion engines use ions, plasma thrusters[2] use a superheated plasma, and mass drivers[3] accelerate small solid pellets. Others heat up a working fluid (usually hydrogen) and expel it through a nozzle, as in chemical rockets. Heat sources considered include conventional nuclear reactors[4], ground-based lasers[5], and antimatter produced on an industrial scale. However, all of these share the common defect of requiring reaction mass to be thrown away to accelerate and to decelerate. Total change in velocity is limited by the reaction mass carried.

It is thus worthwhile to investigate non-rocket means of propulsion, many of which have a much higher theoretical performance. Gravity-boost flybys, using one of the more interesting results of three-body mechanics, have been used with spectacular success to extend the missions of Voyager 1 and 2 to Uranus and Neptune. Galileo, using the experience gained from the Voyagers, is taking a complicated trajectory to get to Jupiter, involving several flybys of Venus and Earth to acquire the necessary energy. The use of gravity assist maneuvers is tightly constrained by alignment of the bodies in question (the alignment of the outer planets which allowed the Voyager 'Grand Tour' occurs only once every century or two); the necessary time (Galileo will require several years to get to Jupiter, instead of the months necessary for a direct mission); and the uncertainty and complexity of multiple flybys.

Launch loops[6] magnetically accelerate an extremely large iron loop inside its static track over a period of time, and then magnetically transfer the stored energy to a launch vehicle over a short time. Electromagnetic catapults, like mass drivers, are essentially linear electric motors. Both of these show great promise but, as launchers, require some other propulsion source after launch. They also require advanced, complex control systems.

Photons carry momentum, and the pressure of sunlight exerts a force on objects. This force is insignificant on objects of ordinary dimensions, but the force on a solar sail[7] many kilometers across and only thousands to tens of molecules thick is considerable. Developing a material that retains the requisite strength when formed into such dimensions is an as yet unsolved engineering problem. In addition, the intensity of sunlight falls off as the square of the distance from the Sun, making solar sails impractical in the outer solar system without giant booster lasers. In addition, solar sails can provide only low acceleration (though the total change in velocity is unlimited.)

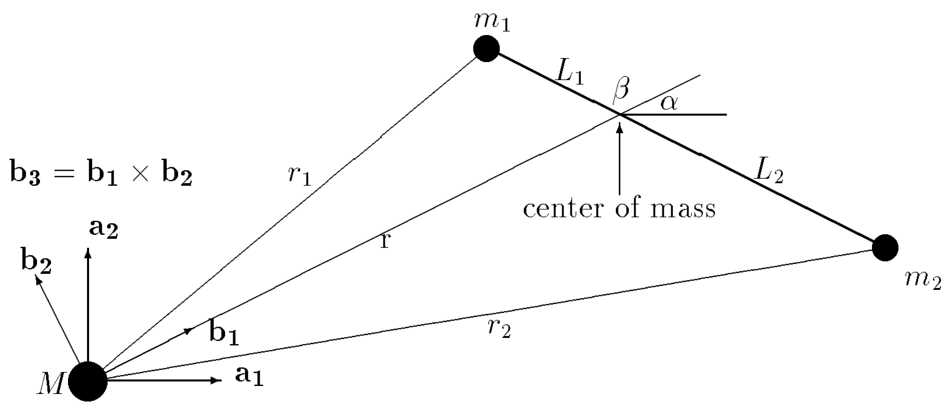
Recently, interest has been growing in the use of tethers, cables tens to hundreds of kilometers long, connected to orbiting spacecraft, for space propulsion and upper-atmospheric and magnetospheric research. In research, they would allow instruments to operate at altitudes too high for atmospheric craft to operate but where air resistance does not allow spacecraft to orbit for long. More exciting, however, is their potential to propel spacecraft, by two different methods. One[8] involves reeling out cable between two parts of a compound satellite, dividing the orbital energy between the parts, to put one in a higher orbit and the other in a lower orbit. The other[9] uses the Earth or other planet's magnetic field as the stator, and a conductive, insulated tether with electrodes on the ends as the rotor of an electric motor. The prime virtue of tethers is their relative simplicity, and their most complicated design problem is their dynamics.

*Chapter 2*

# Lagrangian of Mass Dipole and Equations of Motion

Typically, a tether is a 10-400 km long cable connected to a satellite in orbit around the Earth or some other body. The other end is usually connected to a secondary mass, but may hang free. In what ways might such a system move? The dynamical analysis of a flexible, massive object moving in a gravitational field is exceedingly complicated, so we will first consider the motion of a mass dipole, two masses, $m_1$ and $m_2$ rigidly connected by a massless bar of length $L$, moving in the gravitational field of a point mass. (See Figure 2.1.)

Figure 2.1: Mass Dipole




Frame of reference A is inertial, and $r$ and $\alpha$ are the polar coordinates of the center of mass of the dipole in frame A. Frame B rotates with respect to A such that the center of mass of the dipole is always on the $b_1$ axis. The (vector) angular velocity of frame B in frame A is ${}^A\omega^B$. Superscripts before a derivative indicate in which frame the derivative is to be taken, e.g. $\frac{{}^B d}{dt}r$ is the first derivative of r in reference frame B.

${}^A\mathbf{v}_{c.m.}$ is the vector velocity of the dipole center of mass in the inertial frame

A:

$$\begin{aligned} {}^A\mathbf{v}_{c.m.} &\equiv \frac{{}^A d}{dt} r\mathbf{b_1} \\ &= \frac{{}^B d}{dt} r\mathbf{b_1} + ({}^A\omega^B \times r\mathbf{b_1}) \end{aligned}$$

and since ${}^A\omega^B = \dot{\alpha}\mathbf{b_3}$,

$$\begin{aligned} {}^A\mathbf{v}_{c.m.} &= \dot{r}\mathbf{b_1} + (\dot{\alpha}\mathbf{b_3} \times r\mathbf{b_1}) \\ &= \dot{r}\mathbf{b_1} + r\dot{\alpha}\mathbf{b_2} \end{aligned}$$

$T_{c.m.}$ is the the kinetic energy in $A$ of the whole dipole moving around the origin at the velocity of the center of mass:

$$m \equiv m_1 + m_2$$

$$\begin{aligned} T_{c.m.} &= \frac{1}{2} m ({}^A\mathbf{v_{c.m.}})^{\mathbf{2}} \\ &= \frac{1}{2} m [\dot{r}^2 + r^2 \dot{\alpha}^2] \end{aligned}$$

$T_1$ and $T_2$ are the kinetic energies of the two dipole masses moving around their center of mass:

$$T_1 = \frac{1}{2} m_1 L_1^2 (\dot{\alpha} + \dot{\beta})^2$$

$$T_2 = \frac{1}{2} m_2 L_2^2 (\dot{\alpha} + \dot{\beta})^2$$

The total kinetic energy of the dipole is simply:

$$\begin{aligned} T &= T_{c.m.} + T_1 + T_2 \\ &= \frac{1}{2} m (\dot{r}^2 + r^2\dot{\alpha}^2) + \frac{1}{2}(m_1 L_1^2 + m_2 L_2^2)(\dot{\alpha} + \dot{\beta})^2 \end{aligned}$$

Recognizing $(m_1 L_1^2 + m_2 L_2^2)$ as the moment of inertia about the center of mass and denoting it I, we get

$$T = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\alpha}^2) + \frac{1}{2} I (\dot{\alpha} + \dot{\beta})^2 \tag{2.1}$$

The potential energy, U, is simply the sum of $U_1$, the gravitational potential energy of mass 1 and $U_2$, the gravitational potential energy of mass 2.

$$U_1 = \frac{-GMm_1}{r_1}$$

where G is the gravitational constant and M is the mass of the primary. By the law of cosines we have

$$U_1 = \frac{-GMm_1}{\sqrt{r^2 - 2rL_1 \cos(\pi - \beta) + L_1^2}}$$

Since $\cos(\pi - \beta) = -\cos\beta$

$$U_1 = \frac{-GMm_1}{\sqrt{r^2 + 2rL_1 \cos\beta + L_1^2}} \tag{2.2}$$

Similarly for $U_2$ we have

$$U_2 = \frac{-GMm_2}{\sqrt{r^2 - 2rL_2 \cos\beta + L_2^2}} \tag{2.3}$$

Thus, the Lagrangian is

$$\mathcal{L} = T - U_1 - U_2$$

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\alpha}^2) + \frac{1}{2}I(\dot{\alpha} + \dot{\beta})^2 + \frac{GMm_1}{\sqrt{r^2 + 2rL_1 \cos\beta + L_1^2}} + \frac{GMm_2}{\sqrt{r^2 - 2rL_2 \cos\beta + L_2^2}} \tag{2.4}$$

From the Lagrangian we get the equations of motion:

$$\frac{\partial\mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{r}} = 0$$

$$m\dot{\alpha}^2 r - \frac{GMm_1(r + L_1\cos\beta)}{(r^2 + 2rL_1\cos\beta + L_1^2)^{3/2}} - \frac{GMm_2(r - L_2\cos\beta)}{(r^2 - 2rL_2\cos\beta + L_2^2)^{3/2}} - m\ddot{r} = 0 \tag{2.5}$$

$$\frac{\partial\mathcal{L}}{\partial\alpha} - \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\alpha}} = 0$$

$$0 - \frac{d}{dt}(mr^2\dot{\alpha} + I\dot{\alpha} + I\dot{\beta}) = 0$$

$$-m(2r\dot{r}\dot{\alpha} + r^2\ddot{\alpha}) - I(\ddot{\alpha} + \ddot{\beta}) = 0 \tag{2.6}$$

$$\frac{\partial\mathcal{L}}{\partial\beta} - \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\beta}} = 0$$

$$\frac{GMm_1(rL_1\sin\beta)}{(r^2+2rL_1\cos\beta+L_1^2)^{3/2}} + \frac{GMm_2(-rL_2\sin\beta)}{(r^2-2rL_2\cos\beta+L_2^2)^{3/2}} - \frac{d}{dt}(I\dot{\alpha}+I\dot{\beta}) = 0$$

$$\frac{GMm_1(rL_1\sin\beta)}{(r^2+2rL_1\cos\beta+L_1^2)^{3/2}} + \frac{GMm_2(-rL_2\sin\beta)}{(r^2-2rL_2\cos\beta+L_2^2)^{3/2}} - I(\ddot{\alpha}+\ddot{\beta}) = 0 \qquad (2.7)$$

The three equations of motion, 2.5, 2.6 and 2.7 are complicated enough that we cannot find a general solution for them. We must identify possible classes of solutions and see if they work.

*Chapter 3*

# A Radial Solution to the Equations of Motion

Intuition leads us to suspect that one stable orientation of the tether involves the tether radially aligned such that the line from one tether mass to the other also runs through the primary mass, i.e. either ($\beta = 0$ or $\beta = \pi$), and thus

$$\cos\beta = \pm 1 \tag{3.1}$$

Is this consistent with the equations of motion? If so, what do the equations of motion reduce to? Substituting in Equation 3.1 reduces Equation 2.5 to

$$m\dot{\alpha}^2 r - \frac{GMm_1(r+L_1)}{((r+L_1)^2)^{3/2}} - \frac{GMm_2(r-L_1)}{((r-L_2)^2)^{3/2}} - m\ddot{r} = 0$$

$$m\dot{\alpha}^2 r - \frac{GMm_1}{(r+L_1)^2} - \frac{GMm_2}{(r-L_2)^2} - m\ddot{r} = 0 \tag{3.2}$$

Equation 2.6 reduces to

$$-m(2r\dot{r}\dot{\alpha} + r^2\ddot{\alpha}) - I\ddot{\alpha} = 0 \tag{3.3}$$

Equation 2.7 reduces to

$$0 - 0 - I(\ddot{\alpha} + 0) = 0$$

$$\ddot{\alpha} = 0 \tag{3.4}$$

Since $\dot{\alpha}$ is constant, we define $\omega \equiv \dot{\alpha}$. Substituting equation 3.4 into equation 3.3, we get

$$-m(2r\dot{r}\dot{\alpha} + 0) - 0 = 0$$

$$mr\dot{r}\dot{\alpha} = 0$$

Leaving aside the trivial case where $r = 0$ and the dipole just sits and spins at the origin, either $\dot{r} = 0$ or $\dot{\alpha} = 0$. If we substitute $\dot{\alpha} = 0$ into equation 3.2, we get

$$-\frac{GMm_1}{(r+L_1)^2} - \frac{GMm_2}{(r-L_2)^2} = m\ddot{r}$$

which is the one dimensional equation of a object falling in a gravitational field, i.e. the dipole falls into the primary along a radial line.

If $\dot{r} = 0$, the center of mass moves in a circular orbit around the primary, as do the two masses, at smaller and larger radii. Substituting into equation 3.2 and defining $R \equiv r$ when $\dot{r} = 0$, we get

$$m\dot{\alpha}^2 R - \frac{GMm_1}{(R+L_1)^2} - \frac{GMm_2}{(R-L_2)^2} = 0$$

$$\omega \equiv \dot{\alpha} = \sqrt{\frac{GM}{mR}\left(\frac{m_1}{(R+L_1)^2} + \frac{m_2}{(R-L_2)^2}\right)} \tag{3.5}$$

So for any altitude $R$ of the center of mass, there is a circular orbit, whose angular frequency (and thus period) is given by equation 3.5. This radial solution,

$$\begin{array}{lll} r(t) = R & \dot{r}(t) = 0 & \ddot{r}(t) = 0 \\ \alpha(t) = \omega t & \dot{\alpha}(t) = \omega & \ddot{\alpha}(t) = 0 \\ \beta(t) = 0 & \dot{\beta}(t) = 0 & \ddot{\beta}(t) = 0 \end{array} \tag{3.6}$$

is denoted $S$.

If $L_1 \ll R$ and $L_2 \ll R$ then equation 3.5 reduces to

$$\omega = \sqrt{\frac{GM}{mR}\left(\frac{m_1+m_2}{(R+0)^2}\right)}$$

$$\omega = \sqrt{\frac{GM}{R^3}}$$

which is Kepler's Third Law, for circular orbits. Note that the center of mass does not, in general, orbit at the angular velocity a point mass at its altitude would.

Note that, if $\beta = 0$, the center of mass cannot orbit the primary in a non-circular orbit. As no real satellite can have a perfectly circular orbit, any real tether will have some variation in $\beta$.

The question now is: are solutions close to S stable? Perturbation calculations can be made for the exact equations of motion, but eventually reduce to solving a set of linear, second order, constant coefficient differential equations, with no simple solution. These could be solved, but it is easier to reformulate the Lagrangian and equations of motion as power series expansions, and calculate perturbations to lowest order.

*Chapter 4*

# Series Expansion of the Lagrangian and Equations of Motion

Most of the complication of the equations of motion is in terms which derive from the potential energy. For our expansion to be accurate, we must expand in powers of some characteristic of the system which is fairly small. Hence we define

$$\epsilon \equiv \frac{L}{r}$$

This does not appear directly in the potentials. However, we can pull a factor of $r$ out of the denominator of equation 2.2 to get

$$U_1 = \frac{-GMm_1}{r} \frac{1}{\sqrt{1 + 2(L_1/r)\cos\beta + (L_1/r)^2}} \tag{4.1}$$

To find $L_1/r$ we calculate:

$$\begin{aligned}
m_1 L_1 &= m_2 L_2 \\
m_1 \frac{L_1}{L} &= m_2 \frac{L_2}{L} \\
m_1 \frac{L_1}{L} &= \frac{m_2}{L}(L - L_1) \\
m_1 \frac{L_1}{L} &= m_2 - m_2 \frac{L_1}{L} \\
m_1 \frac{L_1}{L} + m_2 \frac{L_1}{L} &= m_2 \\
m \frac{L_1}{L} &= m_2 \\
L_1 &= L(\frac{m_2}{m}) \\
L_1/r &= \epsilon(\frac{m_2}{m})
\end{aligned}$$

and similarly

$$L_2/r = \epsilon(\frac{m_1}{m})$$

which we can substitute into equation 4.1 to get

$$U_1 = \frac{-GMm_1}{r}\frac{1}{\sqrt{1+2(\epsilon(\frac{m_2}{m}))\cos\beta + (\epsilon(\frac{m_2}{m}))^2}}$$

We can make the binomial expansion of $\sqrt{1+[2(\epsilon(\frac{m_2}{m}))\cos\beta + (\epsilon(\frac{m_2}{m}))^2]}$ to get

$$\begin{aligned} U_1 &= \frac{-GMm_1}{r}\left\{1-\frac{1}{2}[2\epsilon(\frac{m_2}{m})\cos\beta+\epsilon^2(\frac{m_2}{m})^2]+\frac{\frac{1}{2}(\frac{3}{2})}{2}[2\epsilon(\frac{m_2}{m})\cos\beta+\epsilon^2(\frac{m_2}{m})^2]^2+\cdots\right\} \\ &= \frac{-GMm_1}{r}\left\{1-\epsilon(\frac{m_2}{m})\cos\beta+\epsilon^2(\frac{m_2}{m})^2(-\tfrac{1}{2}+\tfrac{3}{2}\cos^2\beta)+\cdots\right\} \end{aligned} \tag{4.2}$$

Similarly

$$U_2 = \frac{-GMm_2}{r}\left\{1+\epsilon(\frac{m_1}{m})\cos\beta+\epsilon^2(\frac{m_1}{m})^2(-\tfrac{1}{2}+\tfrac{3}{2}\cos^2\beta)+\cdots\right\} \tag{4.3}$$

The Langrangian then becomes

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2+r^2\dot{\alpha}^2)+\frac{1}{2}I(\dot{\alpha}+\dot{\beta})^2 \tag{4.4}$$

$$-\frac{GMm_1}{r}\left\{1-\epsilon(\frac{m_2}{m})\cos\beta+\epsilon^2(\frac{m_2}{m})^2(-\tfrac{1}{2}+\tfrac{3}{2}\cos^2\beta)+\cdots\right\} \tag{4.5}$$

$$-\frac{GMm_2}{r}\left\{1+\epsilon(\frac{m_1}{m})\cos\beta+\epsilon^2(\frac{m_1}{m})^2(-\tfrac{1}{2}+\tfrac{3}{2}\cos^2\beta)+\cdots\right\} \tag{4.6}$$

When the terms are combined, the $\epsilon$ terms cancel out. However, terms with $\epsilon^1$ will enter later,so we drop terms with $\epsilon^2$ or higher to get

$$\mathcal{L} = \frac{1}{2}m(\dot{r}^2+r^2\dot{\alpha}^2)+\frac{1}{2}I(\dot{\alpha}+\dot{\beta})^2+\frac{GMm}{r} \tag{4.7}$$

from which we can get approximate equations of motion. To prepare for the next step, we rewrite the equations of motion as

$$\begin{aligned} F_1 &= 0 \\ F_2 &= 0 \\ F_3 &= 0 \end{aligned}$$

$$\begin{aligned} F_1 &= \frac{\partial\mathcal{L}}{\partial r}-\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{r}} \\ &= m\dot{\alpha}^2r-\frac{GMm}{r^2}-m\ddot{r} \end{aligned} \tag{4.8}$$

$$\begin{aligned} F_2 &= \frac{\partial \mathcal{L}}{\partial \alpha} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\alpha}} \\ &= 0 - \frac{d}{dt}[mr^2\dot{\alpha} + I(\dot{\alpha} + \dot{\beta})] \\ &= -m(2r\dot{r}\dot{\alpha} + r^2\ddot{\alpha}) - I(\ddot{\alpha} + \ddot{\beta}) \end{aligned} \tag{4.9}$$

$$\begin{aligned} F_3 &= \frac{\partial \mathcal{L}}{\partial \beta} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\beta}} \\ &= 0 - \frac{d}{dt}[I(\dot{\alpha} + \dot{\beta})] \\ &= -I(\ddot{\alpha} + \ddot{\beta}) \end{aligned} \tag{4.10}$$

To find solutions of the equations of motion near to $S$, we add perturbation functions:

$$\begin{array}{ccc} r(t) = R + \epsilon w_1(t) & \dot{r}(t) = \epsilon \dot{w}_1(t) & \ddot{r}(t) = \epsilon \ddot{w}_1(t) \\ \alpha(t) = \omega t + \epsilon w_2(t) & \dot{\alpha}(t) = \omega + \epsilon \dot{w}_2(t) & \ddot{\alpha}(t) = \epsilon \ddot{w}_2(t) \\ \beta(t) = \epsilon w_3(t) & \dot{\beta}(t) = \epsilon \dot{w}_3(t) & \ddot{\beta}(t) = \ddot{w}_3(t) \end{array} \tag{4.11}$$

We now wish to make a Taylor Series expansion of the first equation of motion around the solution $S$.

$$F_1(r, \dot{r}, \ddot{r}, \alpha, \dot{\alpha}, \ddot{\alpha}, \beta, \dot{\beta}, \ddot{\beta}) = 0$$

$$F_1(S) + \epsilon\,(\cdots) + \frac{\epsilon^2}{2!}\,(\cdots) + \cdots = 0$$

$$\begin{aligned} &F_1(S) + \epsilon \left( \left.\frac{\partial F_1}{\partial r}\right|_S w_1 + \left.\frac{\partial F_1}{\partial \dot{r}}\right|_S \dot{w}_1 + \left.\frac{\partial F_1}{\partial \ddot{r}}\right|_S \ddot{w}_1 + \left.\frac{\partial F_1}{\partial \alpha}\right|_S w_2 + \left.\frac{\partial F_1}{\partial \dot{\alpha}}\right|_S \dot{w}_2 \right. \\ &\left. + \left.\frac{\partial F_1}{\partial \ddot{\alpha}}\right|_S \ddot{w}_2 + \left.\frac{\partial F_1}{\partial \beta}\right|_S w_3 + \left.\frac{\partial F_1}{\partial \dot{\beta}}\right|_S \dot{w}_3 + \left.\frac{\partial F_1}{\partial \ddot{\beta}}\right|_S \ddot{w}_3 \right) \\ &+\frac{\epsilon^2}{2!} \left( \left.\frac{\partial^2 F_1}{\partial r^2}\right|_S w_1^2 + \left.\frac{\partial^2 F_1}{\partial r \partial \dot{r}}\right|_S w_1^2 + \left.\frac{\partial^2 F_1}{\partial r \partial \ddot{r}}\right|_S w_1^2 + \left.\frac{\partial^2 F_1}{\partial r \partial \alpha}\right|_S w_1 w_2 + \left.\frac{\partial^2 F_1}{\partial r \partial \dot{\alpha}}\right|_S w_1 w_2 \right. \\ &+ \left.\frac{\partial^2 F_1}{\partial r \partial \ddot{\alpha}}\right|_S + \left.\frac{\partial^2 F_1}{\partial r \partial \beta}\right|_S w_1 w_3 + \left.\frac{\partial^2 F_1}{\partial r \partial \dot{\beta}}\right|_S w_1 w_3 + \left.\frac{\partial^2 F_1}{\partial r \partial \ddot{\beta}}\right|_S w_1 w_3 + \left.\frac{\partial^2 F_1}{\partial \dot{r}^2}\right|_S \dot{w}_1^2 \\ &\left. + \left.\frac{\partial^2 F_1}{\partial \dot{r} \partial \ddot{r}}\right|_S + \cdots \right) + \cdots = 0 \end{aligned} \tag{4.12}$$

$$
\begin{aligned}
\frac{\partial F_1}{\partial r} &= m\dot{\alpha}^2 + \frac{2GMm}{r^3} \\
\left.\frac{\partial F_1}{\partial r}\right|_S &= m\omega^2 + \frac{2GMm}{R^3} \\
\left.\frac{\partial F_1}{\partial \dot{r}}\right|_S &= 0 \\
\left.\frac{\partial F_1}{\partial \ddot{r}}\right|_S &= -m \\
\left.\frac{\partial F_1}{\partial \alpha}\right|_S &= 0 \\
\left.\frac{\partial F_1}{\partial \dot{\alpha}}\right|_S &= 2mR\omega \\
\left.\frac{\partial F_1}{\partial \ddot{\alpha}}\right|_S &= 0 \\
\frac{\partial F_1}{\partial \beta} &= 0 \\
\left.\frac{\partial F_1}{\partial \beta}\right|_S &= 0 \\
\left.\frac{\partial F_1}{\partial \dot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial F_1}{\partial \ddot{\beta}}\right|_S &= 0
\end{aligned}
$$

Many of the second derivatives are zero, so we will use $x$ to denote $r$, $\alpha$, $\beta$, or any of their derivatives. Since $\mathcal{L}$ has no term of order $\epsilon$, neither do the derivatives of $F_1$:

$$
\begin{aligned}
\frac{\partial^2 F_1}{\partial r^2} &= \frac{-6GMm}{r^4} \\
\left.\frac{\partial^2 F_1}{\partial r^2}\right|_S &= \frac{-6GMm}{R^4} \\
\left.\frac{\partial^2 F_1}{\partial r \partial \dot{r}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial r \partial \ddot{r}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial r \partial \alpha}\right|_S &= 0
\end{aligned}
$$

$$\begin{aligned}
\left.\frac{\partial^2 F_1}{\partial r \partial \dot{\alpha}}\right|_S &= 2m\omega \\
\left.\frac{\partial^2 F_1}{\partial r \partial \ddot{\alpha}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial r \partial \beta}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial r \partial \dot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial r \partial \ddot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \dot{r} \partial x}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \ddot{r} \partial x}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \alpha \partial x}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \dot{\alpha}^2}\right|_S &= 2mR \\
\left.\frac{\partial^2 F_1}{\partial \dot{\alpha} \partial \ddot{\alpha}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \dot{\alpha} \partial \beta}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \dot{\alpha} \partial \dot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \dot{\alpha} \partial \ddot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \ddot{\alpha} \partial x}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \beta \partial x}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \dot{\beta} \partial x}\right|_S &= 0 \\
\left.\frac{\partial^2 F_1}{\partial \ddot{\beta} \partial x}\right|_S &= 0
\end{aligned}$$

Substituting the partials into Equation 4.12 we get

$$F_1(S) + \epsilon\left[\left(m\omega^2 + \frac{2GMm}{R^3}\right) w_1 - m\ddot{w}_1 + 2mR\omega\dot{w}_2\right] + \frac{\epsilon^2}{2!}\left[\frac{-6GMm}{R^4}w_1^2 + 2m\omega w_1\dot{w}_2 + 2mR\dot{w}_2^2\right] = 0$$

Since $F_1(S) = 0$, we can drop that term and factor out an $\epsilon$

$$\left(m\omega^2 + \frac{2GMm}{R^3}\right) w_1 - m\ddot{w}_1 + 2mR\omega\dot{w}_2 + \epsilon\left[\frac{-3GMm}{2R^4}w_1^2 + m\omega w_1\dot{w}_2 + mR\dot{w}_2^2\right] = 0 \tag{4.13}$$

Similarly, for $F_2$ we have

$$F_2(S) + \epsilon(\cdots) + \frac{\epsilon^2}{2!}(\cdots) + \cdots = 0$$

The partials are, to first order:

$$\begin{aligned}
\frac{\partial F_2}{\partial r} &= -2m\dot{r}\dot{\alpha} - 2mr\ddot{\alpha} \\
\left.\frac{\partial F_2}{\partial r}\right|_S &= 0 \\
\left.\frac{\partial F_2}{\partial \dot{r}}\right|_S &= -2mR\omega \\
\left.\frac{\partial F_2}{\partial \ddot{r}}\right|_S &= 0 \\
\left.\frac{\partial F_2}{\partial \alpha}\right|_S &= 0 \\
\frac{\partial F_2}{\partial \dot{\alpha}} &= -2mr\dot{r} \\
\left.\frac{\partial F_2}{\partial \dot{\alpha}}\right|_S &= 0 \\
\left.\frac{\partial F_2}{\partial \ddot{\alpha}}\right|_S &= -mR^2 - I \\
\left.\frac{\partial F_2}{\partial \beta}\right|_S &= 0 \\
\left.\frac{\partial F_2}{\partial \dot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial F_2}{\partial \ddot{\beta}}\right|_S &= -I
\end{aligned}$$

For $F_2$, second and higher derivatives come in with $\epsilon^2$ or higher, so we drop them now, making the second equation of motion

$$\epsilon(-2mR\omega\dot{w}_1 + (-mR^2 - I)\ddot{w}_2 - I\ddot{w}_3) = 0 \tag{4.14}$$

For $F_3$ we have

$$\begin{aligned}
\left.\frac{\partial F_3}{\partial r}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \dot{r}}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \ddot{r}}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \alpha}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \dot{\alpha}}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \ddot{\alpha}}\right|_S &= -I \\
\left.\frac{\partial F_3}{\partial \beta}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \dot{\beta}}\right|_S &= 0 \\
\left.\frac{\partial F_3}{\partial \ddot{\beta}}\right|_S &= -I
\end{aligned}$$

Thus the third equation of motion, to first order, is

$$\epsilon(-I\ddot{w}_2 - I\ddot{w}_3) = 0 \tag{4.15}$$

We now collect the three equations of motion and drop terms with $\epsilon^2$ or higher, so we can solve these differential equations for $w_1$,$w_2$,and $w_3$.

$$\left(m\omega^2 + \frac{2GMm}{R^3}\right) w_1 - m\ddot{w}_1 + 2mR\omega\dot{w}_2 = 0$$

or dropping the factor of $m$

$$\left(\omega^2 + \frac{2GM}{R^3}\right) w_1 - \ddot{w}_1 + 2R\omega\dot{w}_2 \tag{4.16}$$

We collect the terms with a factor of $I$ in the second equation

$$-2mR\omega\dot{w}_1 - mR^2\ddot{w}_2 - I(\ddot{w}_2 + \ddot{w}_3) = 0 \tag{4.17}$$

The third equation becomes

$$-I(\ddot{w}_2 + \ddot{w}_3) = 0 \tag{4.18}$$

We substitute 4.18 into 4.17 to get

$$2R\omega\dot{w}_1 + R^2\ddot{w}_2 = 0$$

$$2\omega\int dw_1 + R\int d\dot{w}_2 = 0$$

$$2\omega w_1 + R\dot{w}_2 + C = 0$$

$$\dot{w}_2 = (-2\omega w_1 - C)/R \tag{4.19}$$

which we can substitute into 4.16

$$\left(\omega^2 + \frac{2GM}{R^3}\right) w_1 - \ddot{w}_1 + 2R\omega\frac{-2\omega w_1 - C}{R} = 0$$

$$\left(3\omega^2 - \frac{2GM}{R^3}\right) w_1 + \ddot{w}_1 - 2\omega C = 0 \tag{4.20}$$

Two solutions of equation 4.20 are $\sin\left(t\sqrt{3\omega^2 - \frac{2GM}{R^3}}\right)$ and $\cos\left(t\sqrt{3\omega^2 - \frac{2GM}{R^3}}\right)$, so the general soulution is

$$w_1 = C_1 \sin\left(t\sqrt{3\omega^2 - \frac{2GM}{R^3}}\right) + C_2 \cos\left(t\sqrt{3\omega^2 - \frac{2GM}{R^3}}\right) \tag{4.21}$$

By equation 4.19, the general solution for $w_2$ is also sinusoidal. By equation 4.18, $w_3$ is sinusoidal as well. Therefore,

> **If $r$, $\alpha$ and $\beta$ are close to the radial solution $S$, the system will stably oscillate around $S$, to first order approximation.**

McCoy[9] states, more generally, that the radial alignment of a flexible tether is stable in three dimensions. These oscillations can be parametrically damped or stimulated[8] by reeling in or reeling out the cable at the proper moments.

*Chapter 5*

# Tethers for Momentum Transfer

How can tethers be used for propulsion? When the tether is radially aligned, in a circular orbit, the whole tether moves at $\omega$, the angular velocity of the the center of mass. This is the angular velocity necessary to maintain a circular orbit only for one point along the tether. The outer mass ($m_1$ if $\beta = 0$) is moving faster than orbital velocity for its altitude and the inner mass is moving slower than orbital velocity for *its* altitude. If the tether is cut, the outer mass will go into a higher orbit, with its perigee at its release point and its apogee on the far side of its orbit. The inner mass will go into a lower orbit with its *apogee* at the release point. The total energy of the tether system is not increased, but divided between the parts of the spacecraft. The gain comes from using orbital energy that would otherwise be wasted. In a conventional rocket launch from Earth, the last stage of the rocket arrives with the satellite in its destination orbit. The orbital energy of this stage, and of stages dropped off along the way is wasted.

With a tether, the mission profile goes as follows: a high acceleration rocket stage or stages puts the spacecraft in low orbit. A tether is unreeled between the payload and the spent booster. When fully unreeled, the tether is cut, putting the payload in a higher orbit and the booster in a lower orbit from which it eventually reenters. A small, final, low acceleration rocket stage attached to the payload adjusts the payload's orbit to the desired one. Most studies of this tether technique have focussed on using the Space Shuttle as the booster. Satellites launched from the Shuttle must use booster stages to reach high orbit. Existing low orbit to high orbit boosters could carry more payload to a given orbit if given a start on a tether. Bekey and Penzo[8] calculate that PAM-A's and PAM-D's (Payload Assist Modules) could carry 60% more payload to geostationary orbit with a 360km tether and Centaurs 20% more. If the Shuttle is used, care will be necessary to ensure the Shuttle retains an acceptable orbit after release.

How will the two tether bodies move after release? If the mass of the tether is small compared to the two bodies, they will take Keplerian orbits based on their positions and velocity at release. (The tether itself is much less stable than when anchored at both ends, but will tend to move to minimise its potential energy by aligning itself radially.) Pogorelov[10] gives equations for the orbital parameters of a point mass, based on its initial position in (two-dimensional) polar coordinates ($r$ and $\eta$), the magnitude of its velocity, $v$, and the angle of its velocity above the local horizontal plane, $\theta$

This technique would become more useful if the tether was permanently attached to a massive object that could act as a store of momentum, such as a space station. Payloads would be boosted up from Earth on a suborbital or low orbital trajectory, caught on the end of a tether extending down from the station, haul themselves up the cable, winch themselves down a cable extending outward from the station, and be released. The energy given to the payloads could come either from high-efficiency reaction engines on the station (a more convenient location than on each individual payload) or from payloads returning to Earth, which would be lowered down the inward tether and released. If a Space Shuttle leaving a space station was first lowered inward on a tether before releasing, the Shuttle would require less fuel to deorbit and the station would gain a much needed boost. If, when the Shuttle arrived, it was hauled up an inward tether, there would be no net gain for the station, but the Shuttle would require less fuel to get to orbit. Space stations require large quantities of supplies to be hauled up to them. Most of the supplies eventually become waste that could be lowered down a tether to reclaim some of the energy used to boost it into orbit. Mars has two moons which have a low enough surface gravity to make them practical as tether anchors[8]. In fact, payloads released from a 6,100 km tether extending outward from Deimos would have escape velocity from Mars. The great mass of these moons would make momentum balancing unnecessary. For a tether to catch a mass in a lower orbit, the rendezvous between the cable end and the payload must be accomplished in a relatively brief period while their velocity and position match.

The tether need not be stationary at the radial position on release. If the tether is rotating or oscillating such that outer mass is moving forward at release, its rotational velocity adds to its velocity on release, and the lower mass is similarly retarded. The effect is similar to using a longer tether, but the final orbits are more eccentric. It also requires a stronger, heavier cable.

This bring us to the question of how strong and heavy a cable is required. In the radial orientation, the tension on the cable is the the difference between the gravitational force on one mass and the centripetal force necessary to keep it in a circular orbit at its altitude, for either mass. The centripetal acceleration of body 1 is $c_1$ and the gravitational force on body 1 is $G_1$, and $\epsilon = L/R$ as before.

$$\begin{aligned}
T &= m_1 c_1 - G_1 \\
&= m_1 r_1 \omega^2 - \frac{GMm_1}{(R+L_1)^2} \\
&= m_1(R+L_1)\frac{GM}{mR}\left(\frac{m_1}{(R+L_1)^2} + \frac{m_2}{(R-L_2)^2}\right) - \frac{GMm_1}{(R+L_1)^2} \\
&= m_1\left(1+\frac{m_2}{m}\frac{L}{R}\right)\frac{GM}{m}\left(\frac{m_1}{R^2\left(1+\frac{m_2}{m}\frac{L}{R}\right)^2} + \frac{m_2}{R^2\left(1-\frac{m_1}{m}\frac{L}{R}\right)^2}\right) - \frac{GMm_1}{R^2\left(1+\frac{m_2}{m}\frac{L}{R}\right)^2} \\
&= m_1\left(1+\frac{m_2}{m}\epsilon\right)\frac{GM}{m}\left(\frac{m_1}{R^2\left(1+\frac{m_2}{m}\epsilon\right)^2} + \frac{m_2}{R^2\left(1-\frac{m_1}{m}\epsilon\right)^2}\right) - \frac{GMm_1}{R^2\left(1+\frac{m_2}{m}\epsilon\right)^2}
\end{aligned}$$

Taking the binomial expansion of $\left(1+\frac{m_2}{m}\epsilon\right)^{-2}$ and $\left(1-\frac{m_2}{m}\epsilon\right)^{-2}$ we get

$$\begin{aligned}
T &= m_1\left(1+\frac{m_2}{m}\epsilon\right)\frac{GM}{m}\left(\frac{m_1}{R^2}[1-2(\frac{m_2}{m})\epsilon] + \frac{m_2}{R^2}[1+2(\frac{m_1}{m})\epsilon]\right) - \\
&\quad \frac{GMm_1}{R^2}[1-2(\frac{m_2}{m})\epsilon] \\
&= \frac{GMm_1}{R^2}\left[(\frac{m_1}{m}) - 2\frac{m_1 m_2 \epsilon}{m^2} + (\frac{m_2}{m}) + 2\frac{m_1 m_2 \epsilon}{m^2} + \right. \\
&\quad \left. \frac{m_1 m_2 \epsilon}{m^2} + \frac{m_2^2 \epsilon}{m^2} - 1 + 2\frac{m_2 \epsilon}{m}\right] \\
&= \frac{GMm_1}{R^2}\left[\left((\frac{m_1}{m}) + (\frac{m_2}{m}) - 1\right) + \epsilon\left(\frac{m_1 m_2}{m^2} + \frac{m_2^2}{m^2} + 2\frac{m_2}{m}\right)\right] \\
&= \frac{GMm_1 m_2}{R^2 m}\left[\left(\frac{m}{m} - 1\right) + \epsilon\left(\frac{m_1}{m} + \frac{m_2}{m} + 2\right)\right] \\
&= \frac{GMm_1 m_2}{R^2 m}[3\epsilon]
\end{aligned}$$

$$= \frac{GMm_1m_2}{R^2m}\frac{3L}{R}$$

The tension is positive, as expected, so the tether is under tension and not compression. Thus, in the radial orientation the tether can be a cable rather than a rigid bar. Bekey and Penzo report that this is true for spinning tethers as well[8].

For the relatively large case of two ten tonne satellites connected by a 100km tether orbiting the Earth with the center of mass at 620km altitude, the tension is only 1740N, which could be supported by five strands of 100-pound fishing line.

# *Chapter 6*

# Electrodynamic Tethers

Most planets have significant, though small, magnetic fields, through which satellites move very quickly. Can we use the planetary magnetic field as the pole pieces and a conductive tether as the rotor of a gigantic electric motor/generator? Planetary magnetic fields are essentially uniform over the size of any feasible wire loop. Since a current loop moving in a uniform field cannot gain any net velocity (though it can acquire angular momentum) we look to the plasma trapped by the planetary magnetic field to complete the current path through the tether. The total momentum of tether + plasma remains constant, but the tether receives a boost.

We first consider an insulated, conducting tether with electrodes on each end, and no independent voltage source. Near the equatorial plane, the magnetic field of most planets is essentially parallel to the planetary surface and points north-south. The tether is aligned in the $\hat{r}$ direction, moves in the $\hat{\alpha}$ direction, and the magnetic field points in the $\hat{\phi}$ direction. (See figure 6.1.) The force on the charges in the wire is given by

$$\vec{F} = Q\vec{v} \times \vec{B}$$

$$\vec{F} = Qv\hat{\alpha} \times B\hat{\phi}$$

$$\vec{F} = QvB\hat{r}$$

so a charge is forced outward along the tether and the electrodes allow the circuit to be completed through the plasma. The power will dissipate in the resistance of the tether and plasma, or in a load connected in series with them. The force on a segment of a current loop is

$$\vec{F} = \int (\vec{I} \times \vec{B}) dl \qquad (6.1)$$

where $\vec{I}$ is the current, $\vec{B}$ is the magnetic field, and $dl$ is the length element. $I$ positive indicates current flowing *outward*. The magnetic drag on a tether of length

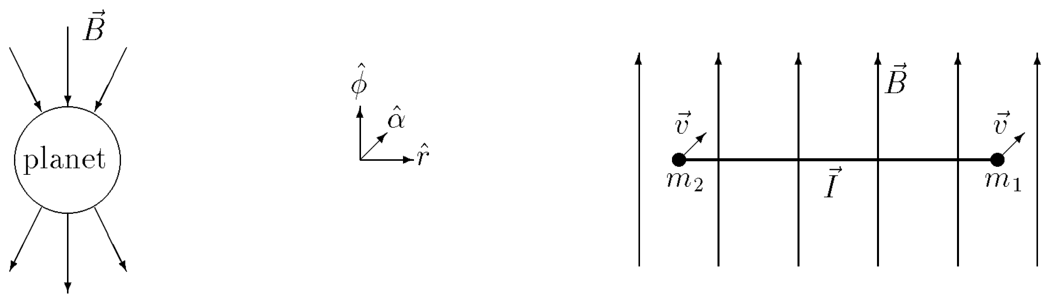
$L$ is

$$\vec{F} = (I\hat{r} \times B\hat{\phi})L$$

$$\vec{F} = -IBL\hat{\alpha}$$

Thus it opposes the motion of the tether, as expected. The tether, planetary magnetic field, and plasma form a generator.

Figure 6.1: Electrodynamic Tether




If the tether is in an orbit with a significant inclination, the tether will have a velocity component in the $\hat{\phi}$ direction. This component of the velocity induces no current, so the magnetic drag and power generated are reduced. The tether will also usually be in a region where $\vec{r}$ and $\vec{B}$ are not perpendicular, so the force on the charges will not be parallel to the wire and the induced emf will be lower, along with the drag force.

If an independently powered voltage source is connected to oppose the induced emf and overcome it, $I$ becomes negative, and the 'drag' force acts to accelerate the tether in its direction of motion. Such a continuous force will gradually raise the altitude of the tether. The tether system is now a motor, turning electrical energy into orbital energy — we have propulsion without reaction mass.

We now consider whether the magnetospheric plasma can carry the necessary currents. The plasma is confined to primarily move along the magnetic field lines, but can move between lines when they become closely packed near the magnetic poles. Parker[12] gives an approximate expression for the conductivity $\sigma$ of a hydrogen plasma, when the ionization is above a few percent (hydrogen being the principal constituent):

$$\sigma \approx 10^7 T^{3/2} \mathrm{sec}^{-1} \tag{6.2}$$

where $T$ is the absolute temperature. The temperature of the exospheric plasma varies with altitude, time of day, and solar activity, but is almost always between 500 and 3000K[13] and usually above 1000K. At 1000K, the conductivity works out to be $3 \cdot 10^{11} \mathrm{sec}^{-1}$ or, in MKS units, a resistivity of $3.0 \cdot 10^6 \mu\Omega\,\mathrm{cm}$ which at first seems impossibly high relative to conductors such as aluminum at $2.824 \mu\Omega\,\mathrm{cm}$[14], but because the current spreads throughout a large volume, essentially all the resistance is near the electrodes and the actual resistance of the plasma as a circuit element is determined by their configuration. The electrodes must make electrical contact between the tether and the plasma it is moving through at orbital velocity. The simplest electrodes are metallic balloons which ensure sufficient contact through surface area. The resistance between a pair of spheres of radius $a$ immersed in a conducting medium of resistivity $\rho$ is simply double the resistance between one sphere and infinity:

$$R = 2\frac{\rho}{4\pi a}$$

For 1m spheres in a medium of resistivity $3.0 \cdot 10^6 \mu\Omega\,\mathrm{cm}$ the resistance is $.0048\Omega$. However, the electrodes are not stationary in a continuous conductive medium but rather are moving through a cloud of changed particles with their own (non-orbital) trajectories. The electrodes and plasma are therefore not a linear resistor, but a current-limited circuit element, with the allowable current determined by the surface area of the balloons. In fact, large enough balloons are needed that it pays to look for more efficient electrodes, with smaller surface areas and less air resistance. One possibility is to replace the negative balloon by an electron gun. A third possibility used but not explained by McCoy[9] is "hollow cathode plasma emitters" developed for ion rockets. Since a conductive tether can act as both a motor and generator, it can store energy as kinetic energy and reconvert it to electrical energy. This is most applicable to solar power satelites and solar powered satelites. Energy is produced at a rate independant of demand, unlike fuel-based generators. A tether would allow storage of unused power and reconversion at times of high demand and while the satelite is shadowed by the Earth. The solar panels have a large surface area and lange air resistance. Diverting a fraction of the electric power to a tether could provide the rebost neccesary. A planetary probe, to the Jovian moons for example (Jupiter having a large magnetic field and many moons), equiped with solar panels and an electrodynamic tether could manuver indefinitely, from moon to moon, until it broke down or wore out.

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