15-859(D) Randomized Algorithms

Homework # 4 **due: October 29, 1998**

Problems:

1. Let G be a bipartite graph with n nodes on the left and n nodes on the right such that each node on the left has degree 2. Prove that for any constant $\epsilon > 0$, there will exist (for sufficiently large n), some set S of $O(\log n)$ vertices on the left such that $|N(S)| \leq (1 + \epsilon)|S|$. In other words, G is not an expander.

   Hint: think of an n-node graph H with one vertex for each right-hand node of G, and an edge for each left-hand node of G.

2. Let G be an n-node cycle, and let $M(G)$ be the matrix for a random walk on G where in each step, with probability 1/4 you move left, with probability 1/4 you move right, and with probability 1/2 you stay put. The eigenvector of largest eigenvalue is the all-ones vector. The eigenvector of smallest eigenvalue (0) has alternating $+1$ and -1's. Describe what one of the other eigenvectors looks like.

   Hint: we want roughly a function such that a weighted average of the function with its second derivative is a multiple of itself.