15-854 Approximation and Online Algorithms

Homework # 3 **due: March 1, 2000**

Groundrules: Same as before.

Exercises:

1. Show how the greedy set cover algorithm can be used to get an $O(\log n)$ approximation for the shortest superstrings problem.

2. In class we saw how SDPs can be used in an approximation algorithm for coloring 3-colorable graphs. We examined two methods for "rounding" the SDP: a simpler one using random hyperplanes through the origin, and a better but more complicated method that used points picked from an n-dimensional gaussian.

   What does the simpler version give us for the problem of appriximately coloring *4-colorable* graphs? (One useful fact: if you have 4 equally-spaced unit vectors in 3-d, then their pairwise dot product will be $-1/3$.) You can ignore logarithms and don't need to give the exponent in closed form.

3. Any 3-point metric space can be perfectly embedded in R^2 (all distances preserved exactly). Give an example of a 4-point metric space that cannot be perfectly embedded in R^3 (or even R^4).

Problems:

4. Show that any n-point metric space can be perfectly embedded in an n-dimensional L_∞ norm. In the L_∞ norm, the distance between two points x and y is $\max_i |x_i - y_i|$. For example, the distance between $(1, 2, 3)$ and $(2, 4, 6)$ is 3. Hint: try some simple examples first, like $n = 3$, and remember that the original space must satisfy the triangle inequality.

Extra non-credit. This week's cartalk puzzler is the following. You have two bottles of pills, A and B. Each day you need to take one of each. One day when taking out your pills, two of them fall out of the B bottle. So, you have 3 pills in your hand — one A pill and two B pills — and you can't tell which is which (only the bottles are marked, not the pills). Unfortunately, taking two B pills in the same day is dangerous, and you don't want to throw them out because they are expensive. What to do?