15-854 Approximation and Online Algorithms

Homework # 5 **due: April 10, 2000**

Groundrules: If you are going to work with someone else on the problem, do the exercise by yourself first.

Recall the "searching for a hole in the fence" scenario we discussed in class: you are standing next to a long fence, extending as far as you can see in both directions. You want to cross the fence and you know that somewhere it has a hole you can go through. But, you don't know whether the hole is to your right or left, or how far away it is.

Let's model this problem as follows: you are initially located at the origin on the real line. The hole is at some unknown (positive or negative) integer coordinate h. You can move left or right at cost equal to the distance moved, and the game continues until you reach h. We will consider the goal of minimizing the competitive ratio; i.e., the ratio of the distance traveled to $|h|$.

Here is a deterministic strategy with competitive ratio of 9 (which is optimal for deterministic strategies): move to location 1, then to -2, then to 4, then to -8, etc. The worst case is if $h = 2^{2i} + 1$ for some integer i. In this case, the on-line cost is $(1 + 1) + (2 + 2) + (4 + 4) + \ldots + (2^{2i} + 2^{2i}) + (2^{2i+1} + 2^{2i+1}) + h \; < \; 9|h|$.

Exercises:

1. Describe and analyze a randomized algorithm whose competitive ratio is at most 7. That is, for any h, the expected cost of the algorithm is at most $7|h|$.

Problems:

2. Describe and analyze a randomized algorithm whose competitive ratio is at most $7 - \epsilon$ for some constant $\epsilon > 0$.

(Optimum is 4.511)