The Hungry Lion

A hungry lion runs inside a circus arena which is a circle of radius 10 meters. Running in broken lines (i.e. along a piecewise linear trajectory), the lion covers 30 kilometers. Prove that the sum of all turning angles is at least 2998 radians.

Solution.

Let us look at the world through the eyes of the hungry lion, that is, the lion is stationary but everything else moves around. Assuming furthermore that the poor animal has a stiff neck and cannot rotate its head and is always facing North. The trajectory of the center of the arena looks in the the following way: It is a combination of linear segments pointing South and circular arcs. The total length of the linear segments is 30 km. Since the center is never more than 10 meters away from the lion, the initial and final positions of the center are at most 20 meters apart.

By a form of the triangle inequality, the total length of arcs is at least $30,000 - 20 = 29,980$ meters. But each arc has radius at most 10 meters, so the sum of all arc angles is at least $2,998$ radians.