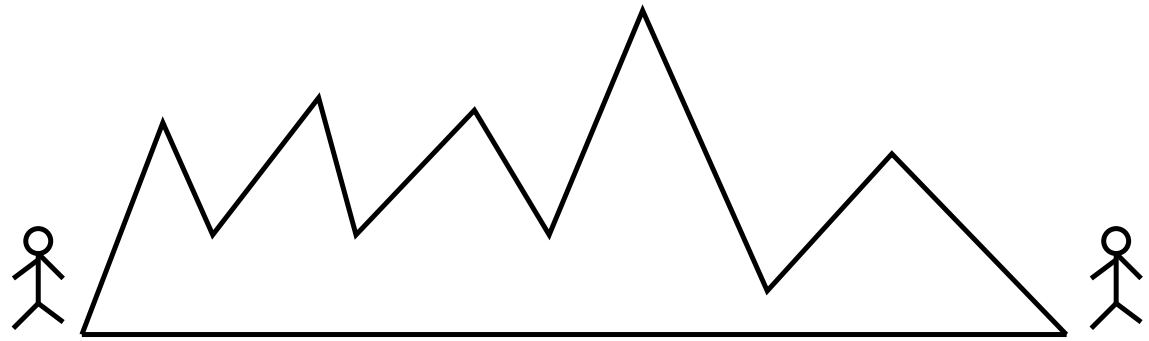
Level with me

Alan and Danny are at opposite ends of a mountain range and need to get together to think of more puzzles. They both start exactly at sea level and want to stay at exactly the same height throughout the journey, even if this means sometimes going backwards? Can it be done?

A mountain range can be represented by a peice-wise linear curve in 2 dimensions. See the diagram below.



Solution

We form a graph in the following manner. Draw a horizontal line through every peak and through every valley of the mountain range as well as one at ground level. Let P be the set of points where these lines intersect the mountain range. The vertices of the graph will consist of pairs of points in P that belong to the same horizontal line together with a distinct vertex for each valley and peak. Two vertices are connected by an edge if and only if the climbers can start at the two points given by one vertex and walk separate monotone paths (always ascending/descending) to the two points of the other vertex. Thus peak and valley vertices have degree 1. If a and b are the start positions of the climbers then the vertex (a, b) has degree 1. A simple checking of cases shows that all other vertices are of degree 0,2, or 4, so the connected component of (a, b) in the graph must contain another vertex of odd degree, namely a peak or a valley vertex. But a path from (a, b) to a peak or a valley in the graph, is by definition exactly what we are looking for.

Note that there may not be a solution if there are points on the range below sea level.

This puzzle was taken from Doug West's book on Graph Theory [1] where it is attributed to D.G. Hoffman.

The above solution was provied by Cliff Smyth. Vince Conitzer, Christopher J Peikert, Mike Schuresko and Kielly E. Yates, all provided solutions.

References

[1] D. West, Introduction to Graph Theory, Prentice Hall, 2001.