Another Hat Problem
Our $2n$ hat wearing friends are once again standing in a circle so that everyone can see everybody else's hat. At a certain moment in time each person must simultaneously shout "my hat is black" or "my hat is white". The team wins a big prize if at least n of the people get the color of their hat right. Of course, if fewer than n get it right, the whole team is eliminated and this is painful. Can they guarantee to avoid being eliminated?

Solution Initially the friends divide themeselves into groups S_0, S_1 of size n. The members of group S_i proceed on the assumption that the number of black hats is equal to i modulo 2, and make their guess accordingly. One group will be correct and they will all guess correctly.

This can be easily be generalised in the following way. Suppose now that we have k colours $0, 1, \ldots, k-1$ and kn people and we still want to guarantee that n people get their hat color correct. Let the *weight* of a hat coloring be the sum of the colors of the hats modulo k. E.g. if there are 3 colors and the hats have colors 1,1,2,0,0,2,1,2,2 then the weight is 1+1+2+0+0+2+1+2+2 modulo 3 = 2. Now the friends divide themeselves into groups $S_0, S_1, \ldots, S_{k-1}$ of size n. The members of group S_i proceed on the assumption that the weight of the coloring is equal to i and make their guess accordingly. One group will be correct and they will all guess correctly.