The Gridville Garbash problem

Gridville is a perfect city. It is laid out as an $n \times n$ grid and each of n^2 families inhabits its own square. A developer offers to buy $k < n$ plots at a price of one billion Wazooli's per plot. If a plot is bought, the family will move out and the plot will be used for growing Garbash, the most valuable commodity in Grid World. If at any time, a family plot has two Garbash plots adjacent to it[1], the smell of the Garbash will cause them to leave and the developer will buy up the plot for a mere million Wazooli's and start growing Garbash. After, 10 years, the developer agrees to clean up and replace the plots by family homes, **unless** everybody has left.

The developer will not disclose where he plans to put his k initial plots. Should the inhabitants of Gridville take the money, given that they want to get back to normal in 10 years?

Solution: This is a question which can be difficult to figure out, but nevertheless has a rather simple elegant solution. The key is to consider the *perimeter* of the region defined by the Garbash plots. Initially this is at most $4k$ in size. Now observe that adding a plot does not increase the perimeter. The new plot has 4 sides and $x \geq 2$ of them already count as perimeter for the current region. When we add the plot we gain $4 - x$ in perimeter and lose x. Since $4 - x \leq x$ we do not increase the perimeter.

This means that the perimeter of the garbash region is always less than 4n, the perimeter of Gridville and so not everybody will leave.

Thanks go to Mike Schuresko and Sunny Daniels for sending in solutions.

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[1] Plot (x, y) is adjacent to plot (x', y') iff $x = x'$ and $|y - y'| = 1$ or $y = y'$ and $|x - x'| = 1$.